Mathematica 11.3 Integration Test Results

Test results for the 153 problems in "5.3.7 Inverse tangent functions.m"

Problem 15: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcTan}\left[\frac{\sqrt{-e}\, x}{\sqrt{d+e x^2}}\right]}{x^2} \, dx$$

Optimal (type 3, 59 leaves, 4 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{-e}\, x}{\sqrt{d+e x^2}}\right]}{x}-\frac{\sqrt{-e}\, \text{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}}\right]}{\sqrt{d}}$$

Result (type 3, 86 leaves):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{-e}\, x}{\sqrt{d+e x^2}}\right]}{x}+\frac{i \sqrt{e}\, \text{Log}\left[\frac{2 i \sqrt{d}}{\sqrt{e}\, x}-\frac{2 \sqrt{-e}\, \sqrt{d+e x^2}}{e x}\right]}{\sqrt{d}}$$

Problem 18: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{9/2}\, \text{ArcTan}\left[\frac{\sqrt{-e}\, x}{\sqrt{d+e x^2}}\right] dx$$

Optimal (type 4, 211 leaves, 6 steps):

$$\frac{60 d^2 \sqrt{x}\, \sqrt{d+e x^2}}{847 (-e)^{5/2}}+\frac{36 d x^{5/2} \sqrt{d+e x^2}}{847 (-e)^{3/2}}+\frac{4 x^{9/2} \sqrt{d+e x^2}}{121 \sqrt{-e}}+\frac{2}{11} x^{11/2}\, \text{ArcTan}\left[\frac{\sqrt{-e}\, x}{\sqrt{d+e x^2}}\right]+$$
$$\left(30 d^{11/4} \sqrt{-e}\, \left(\sqrt{d}+\sqrt{e}\, x\right) \sqrt{\frac{d+e x^2}{\left(\sqrt{d}+\sqrt{e}\, x\right)^2}}\, \text{EllipticF}\left[2\, \text{ArcTan}\left[\frac{e^{1/4} \sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$
$$\left(847 e^{13/4} \sqrt{d+e x^2}\right)$$

Result (type 4, 170 leaves):

$$\frac{4\sqrt{x}\,\sqrt{d+e\,x^2}\,\left(15\,d^2-9\,d\,e\,x^2+7\,e^2\,x^4\right)}{847\,(-e)^{5/2}}+\frac{2}{11}\,x^{11/2}\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]-$$

$$\frac{60\,i\,d^3\,\sqrt{1+\frac{d}{e\,x^2}}\,x\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right],-1\right]}{847\,\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\,(-e)^{5/2}\,\sqrt{d+e\,x^2}}$$

Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{5/2}\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]dx$$

Optimal (type 4, 181 leaves, 5 steps):

$$\frac{20\,d\,\sqrt{x}\,\sqrt{d+e\,x^2}}{147\,(-e)^{3/2}}+\frac{4\,x^{5/2}\,\sqrt{d+e\,x^2}}{49\,\sqrt{-e}}+\frac{2}{7}\,x^{7/2}\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]-$$

$$\left(10\,d^{7/4}\,\sqrt{-e}\,\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+e\,x^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{e^{1/4}\,\sqrt{x}}{d^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(147\,e^{9/4}\,\sqrt{d+e\,x^2}\right)$$

Result (type 4, 158 leaves):

$$\frac{2}{147}\,\sqrt{x}\,\left(\frac{2\,\left(5\,d-3\,e\,x^2\right)\,\sqrt{d+e\,x^2}}{(-e)^{3/2}}+21\,x^3\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]\right)-$$

$$\frac{20\,i\,d^2\,\sqrt{1+\frac{d}{e\,x^2}}\,x\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right],-1\right]}{147\,\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\,(-e)^{3/2}\,\sqrt{d+e\,x^2}}$$

Problem 20: Result unnecessarily involves imaginary or complex numbers.

$$\int\sqrt{x}\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]dx$$

Optimal (type 4, 153 leaves, 4 steps):

$$\frac{4\sqrt{x}\,\sqrt{d+e\,x^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]+\frac{1}{9\,e^{5/4}\sqrt{d+e\,x^2}}$$
$$2\,d^{3/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+e\,x^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 147 leaves) :

$$\frac{4\sqrt{x}\,\sqrt{d+e\,x^2}}{9\sqrt{-e}}+\frac{2}{3}x^{3/2}\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]-\frac{4\,i\,d\sqrt{1+\frac{d}{e\,x^2}}\ x\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right],-1\right]}{9\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\ \sqrt{-e}\ \sqrt{d+e\,x^2}}$$

Problem 21: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]}{x^{3/2}}\,dx$$

Optimal (type 4, 122 leaves, 3 steps) :

$$-\frac{2\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]}{\sqrt{x}}+$$
$$\left(2\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+e\,x^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(d^{1/4}\,e^{1/4}\sqrt{d+e\,x^2}\right)$$

Result (type 4, 115 leaves) :

$$-\frac{2\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]}{\sqrt{x}}+\frac{4\,i\sqrt{-e}\sqrt{1+\frac{d}{e\,x^2}}\ x\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right],-1\right]}{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\ \sqrt{d+e\,x^2}}$$

Problem 22: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]}{x^{7/2}}\,dx$$

Optimal (type 4, 156 leaves, 4 steps) :

$$-\frac{4\sqrt{-e}\sqrt{d+e x^2}}{15 d x^{3/2}}-\frac{2\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e x^2}}\right]}{5 x^{5/2}}-\frac{1}{15 d^{5/4}\sqrt{d+e x^2}}$$

$$2\sqrt{-e}\,e^{3/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+e x^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 150 leaves):

$$-\frac{2\left(2\sqrt{-e}\,x\sqrt{d+e x^2}+3 d\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e x^2}}\right]\right)}{15 d x^{5/2}}+$$

$$\frac{4\,i\,(-e)^{3/2}\sqrt{1+\frac{d}{e x^2}}\,x\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right],-1\right]}{15 d\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+e x^2}}$$

Problem 23: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e x^2}}\right]}{x^{11/2}}\,dx$$

Optimal (type 4, 186 leaves, 5 steps):

$$-\frac{4\sqrt{-e}\sqrt{d+e x^2}}{63 d x^{7/2}}-\frac{20(-e)^{3/2}\sqrt{d+e x^2}}{189 d^2 x^{3/2}}-\frac{2\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e x^2}}\right]}{9 x^{9/2}}+$$

$$\left(10\sqrt{-e}\,e^{7/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+e x^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(189 d^{9/4}\sqrt{d+e x^2}\right)$$

Result (type 4, 162 leaves):

$$\frac{4\sqrt{-e}\,x\sqrt{d+e x^2}\left(-3 d+5 e x^2\right)-42 d^2\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e x^2}}\right]}{189 d^2 x^{9/2}}+$$

$$\frac{20\,i\,(-e)^{5/2}\sqrt{1+\frac{d}{e x^2}}\,x\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right],-1\right]}{189 d^2\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+e x^2}}$$

Problem 24: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcTan}\left[\frac{\sqrt{-e}\, x}{\sqrt{d+e x^2}}\right]}{x^{15/2}} \, dx$$

Optimal (type 4, 216 leaves, 6 steps):

$$-\frac{4 \sqrt{-e} \sqrt{d+e x^2}}{143 d x^{11/2}}-\frac{36 (-e)^{3/2} \sqrt{d+e x^2}}{1001 d^2 x^{7/2}}-\frac{60 (-e)^{5/2} \sqrt{d+e x^2}}{1001 d^3 x^{3/2}}-\frac{2 \text{ArcTan}\left[\frac{\sqrt{-e}\, x}{\sqrt{d+e x^2}}\right]}{13 x^{13/2}}-\left(30 \sqrt{-e}\, e^{11/4} \left(\sqrt{d}+\sqrt{e}\, x\right) \sqrt{\frac{d+e x^2}{\left(\sqrt{d}+\sqrt{e}\, x\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{e^{1/4} \sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(1001 d^{13/4} \sqrt{d+e x^2}\right)$$

Result (type 4, 171 leaves):

$$\frac{1}{1001 x^{13/2}} 2 \left(-\frac{2 \sqrt{-e} \sqrt{d+e x^2} \left(7 d^2 x-9 d e x^3+15 e^2 x^5\right)}{d^3}-77 \text{ArcTan}\left[\frac{\sqrt{-e}\, x}{\sqrt{d+e x^2}}\right]+\frac{30 i (-e)^{7/2} \sqrt{1+\frac{d}{e x^2}}\, x^{15/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right], -1\right]}{d^3 \sqrt{\frac{i \sqrt{d}}{\sqrt{e}}} \sqrt{d+e x^2}}\right)$$

Problem 25: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{7/2} \text{ArcTan}\left[\frac{\sqrt{-e}\, x}{\sqrt{d+e x^2}}\right] dx$$

Optimal (type 4, 326 leaves, 7 steps):

$$\frac{28\,d\,x^{3/2}\,\sqrt{d+e\,x^2}}{405\,(-e)^{3/2}}+\frac{4\,x^{7/2}\,\sqrt{d+e\,x^2}}{81\,\sqrt{-e}}-\frac{28\,d^2\,\sqrt{-e}\,\sqrt{x}\,\sqrt{d+e\,x^2}}{135\,e^{5/2}\,\left(\sqrt{d}+\sqrt{e}\,x\right)}+\frac{2}{9}\,x^{9/2}\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]+$$

$$\left(28\,d^{9/4}\,\sqrt{-e}\,\left(\sqrt{d}+\sqrt{e}\,x\right)\,\sqrt{\frac{d+e\,x^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{e^{1/4}\,\sqrt{x}}{d^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(135\,e^{11/4}\,\sqrt{d+e\,x^2}\right)-$$

$$\left(14\,d^{9/4}\,\sqrt{-e}\,\left(\sqrt{d}+\sqrt{e}\,x\right)\,\sqrt{\frac{d+e\,x^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{e^{1/4}\,\sqrt{x}}{d^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(135\,e^{11/4}\,\sqrt{d+e\,x^2}\right)$$

Result (type 4, 263 leaves):

$$\left(2\,\sqrt{x}\,\left(x\,\sqrt{\frac{i\,\sqrt{e}\,x}{\sqrt{d}}}\right.\right.$$

$$\left(14\,d^2\,\sqrt{-e^2}+4\,d\,\sqrt{-e}\,e^{3/2}\,x^2+10\,\left(-e^2\right)^{3/2}\,x^4+45\,e^{5/2}\,x^3\,\sqrt{d+e\,x^2}\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]\right)-$$

$$42\,d^{5/2}\,\sqrt{-e}\,\sqrt{1+\frac{e\,x^2}{d}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{e}\,x}{\sqrt{d}}}\right],-1\right]+42\,d^{5/2}\,\sqrt{-e}$$

$$\left.\left.\sqrt{1+\frac{e\,x^2}{d}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{e}\,x}{\sqrt{d}}}\right],-1\right]\right)\right)\Big/\left(405\,e^{5/2}\,\sqrt{\frac{i\,\sqrt{e}\,x}{\sqrt{d}}}\,\sqrt{d+e\,x^2}\right)$$

Problem 26: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{3/2}\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]\,dx$$

Optimal (type 4, 296 leaves, 6 steps):

$$\frac{4\,x^{3/2}\,\sqrt{d+e\,x^2}}{25\,\sqrt{-e}}+\frac{12\,d\,\sqrt{-e}\,\sqrt{x}\,\sqrt{d+e\,x^2}}{25\,e^{3/2}\,\left(\sqrt{d}+\sqrt{e}\,x\right)}+\frac{2}{5}\,x^{5/2}\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]-\frac{1}{25\,e^{7/4}\,\sqrt{d+e\,x^2}}$$

$$12\,d^{5/4}\,\sqrt{-e}\,\left(\sqrt{d}+\sqrt{e}\,x\right)\,\sqrt{\frac{d+e\,x^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{e^{1/4}\,\sqrt{x}}{d^{1/4}}\right],\frac{1}{2}\right]+$$

$$\frac{1}{25\,e^{7/4}\,\sqrt{d+e\,x^2}}6\,d^{5/4}\,\sqrt{-e}\,\left(\sqrt{d}+\sqrt{e}\,x\right)\,\sqrt{\frac{d+e\,x^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{e^{1/4}\,\sqrt{x}}{d^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 244 leaves):

$$-\left(\left(2\sqrt{x}\left(x\sqrt{\frac{i\sqrt{e}\,x}{\sqrt{d}}}\left(2\,d\sqrt{-e^2}+2\sqrt{-e}\,e^{3/2}\,x^2-5\,e^{3/2}\,x\sqrt{d+e\,x^2}\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]\right)-6\,d^{3/2}\sqrt{-e}\sqrt{1+\frac{e\,x^2}{d}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{e}\,x}{\sqrt{d}}}\right],-1\right]+6\,d^{3/2}\sqrt{-e}\sqrt{1+\frac{e\,x^2}{d}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{e}\,x}{\sqrt{d}}}\right],-1\right]\right)\right)\Big/\left(25\,e^{3/2}\sqrt{\frac{i\sqrt{e}\,x}{\sqrt{d}}}\sqrt{d+e\,x^2}\right)\right)$$

Problem 27: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]}{\sqrt{x}}\,dx$$

Optimal (type 4, 260 leaves, 5 steps):

$$-\frac{4\sqrt{-e}\sqrt{x}\sqrt{d+e\,x^2}}{\sqrt{e}\left(\sqrt{d}+\sqrt{e}\,x\right)}+2\sqrt{x}\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]+\frac{1}{e^{3/4}\sqrt{d+e\,x^2}}4\,d^{1/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+e\,x^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right],\frac{1}{2}\right]-\frac{1}{e^{3/4}\sqrt{d+e\,x^2}}2\,d^{1/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+e\,x^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 208 leaves):

$$\left(2\sqrt{x}\left(\sqrt{e}\sqrt{\frac{i\sqrt{e}\,x}{\sqrt{d}}}\sqrt{d+e\,x^2}\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]-2\sqrt{d}\sqrt{-e}\sqrt{1+\frac{e\,x^2}{d}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{e}\,x}{\sqrt{d}}}\right],-1\right]+2\sqrt{d}\sqrt{-e}\sqrt{1+\frac{e\,x^2}{d}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{e}\,x}{\sqrt{d}}}\right],-1\right]\right)\right)\Big/\left(\sqrt{e}\sqrt{\frac{i\sqrt{e}\,x}{\sqrt{d}}}\sqrt{d+e\,x^2}\right)$$

Problem 28: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcTan}\left[\frac{\sqrt{-e}\, x}{\sqrt{d+e x^2}}\right]}{x^{5/2}} \, dx$$

Optimal (type 4, 298 leaves, 6 steps):

$$-\frac{4 \sqrt{-e} \sqrt{d+e x^2}}{3 d \sqrt{x}}+\frac{4 \sqrt{-e^2} \sqrt{x} \sqrt{d+e x^2}}{3 d \left(\sqrt{d}+\sqrt{e} x\right)}-\frac{2 \text{ArcTan}\left[\frac{\sqrt{-e}\, x}{\sqrt{d+e x^2}}\right]}{3 x^{3/2}}-\frac{1}{3 d^{3/4} \sqrt{d+e x^2}}$$
$$4 \sqrt{-e}\, e^{1/4} \left(\sqrt{d}+\sqrt{e} x\right) \sqrt{\frac{d+e x^2}{\left(\sqrt{d}+\sqrt{e} x\right)^2}}\ \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{e^{1/4} \sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]+$$
$$\frac{1}{3 d^{3/4} \sqrt{d+e x^2}} 2 \sqrt{-e}\, e^{1/4} \left(\sqrt{d}+\sqrt{e} x\right) \sqrt{\frac{d+e x^2}{\left(\sqrt{d}+\sqrt{e} x\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{e^{1/4} \sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 234 leaves):

$$\left(-2 \sqrt{\frac{i \sqrt{e}\, x}{\sqrt{d}}} \left(2 \sqrt{-e}\, x \left(d+e x^2\right)+d \sqrt{d+e x^2}\ \text{ArcTan}\left[\frac{\sqrt{-e}\, x}{\sqrt{d+e x^2}}\right]\right)+\right.$$
$$4 \sqrt{d} \sqrt{-e^2}\, x^2 \sqrt{1+\frac{e x^2}{d}}\ \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{e}\, x}{\sqrt{d}}}\right], -1\right]-$$
$$\left.4 \sqrt{d} \sqrt{-e^2}\, x^2 \sqrt{1+\frac{e x^2}{d}}\ \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{e}\, x}{\sqrt{d}}}\right], -1\right]\right) \Big/$$
$$\left(3 d x^{3/2} \sqrt{\frac{i \sqrt{e}\, x}{\sqrt{d}}} \sqrt{d+e x^2}\right)$$

Problem 29: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcTan}\left[\frac{\sqrt{-e}\, x}{\sqrt{d+e x^2}}\right]}{x^{9/2}} \, dx$$

Optimal (type 4, 331 leaves, 7 steps):

$$-\frac{4\sqrt{-e}\,\sqrt{d+e\,x^2}}{35\,d\,x^{5/2}}-\frac{12\,(-e)^{3/2}\sqrt{d+e\,x^2}}{35\,d^2\sqrt{x}}-\frac{12\sqrt{-e}\,e^{3/2}\sqrt{x}\,\sqrt{d+e\,x^2}}{35\,d^2\left(\sqrt{d}+\sqrt{e}\,x\right)}-\frac{2\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]}{7\,x^{7/2}}+$$
$$\frac{1}{35\,d^{7/4}\sqrt{d+e\,x^2}}12\sqrt{-e}\,e^{5/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+e\,x^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right],\frac{1}{2}\right]-$$
$$\frac{1}{35\,d^{7/4}\sqrt{d+e\,x^2}}6\sqrt{-e}\,e^{5/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+e\,x^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 256 leaves):

$$\left(2\left(\sqrt{\frac{i\sqrt{e}\,x}{\sqrt{d}}}\left(2\sqrt{-e}\,x\left(-d^2+2\,d\,e\,x^2+3\,e^2\,x^4\right)-5\,d^2\sqrt{d+e\,x^2}\,\text{ArcTan}\left[\frac{\sqrt{-e}\,x}{\sqrt{d+e\,x^2}}\right]\right)+\right.\right.$$
$$6\sqrt{d}\,(-e)^{3/2}\sqrt{e}\,x^4\sqrt{1+\frac{e\,x^2}{d}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{e}\,x}{\sqrt{d}}}\right],-1\right]+6\sqrt{d}\,\sqrt{-e}\,e^{3/2}\,x^4$$
$$\left.\left.\sqrt{1+\frac{e\,x^2}{d}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{e}\,x}{\sqrt{d}}}\right],-1\right]\right)\right)\Big/\left(35\,d^2\,x^{7/2}\sqrt{\frac{i\sqrt{e}\,x}{\sqrt{d}}}\,\sqrt{d+e\,x^2}\right)$$

Problem 32: Unable to integrate problem.

$$\int\frac{\left(a+b\,\text{ArcTan}\left[\frac{\sqrt{1-c\,x}}{\sqrt{1+c\,x}}\right]\right)^3}{1-c^2\,x^2}\,dx$$

Optimal (type 4, 431 leaves, 9 steps):

$$-\frac{2\left(a+b\,\text{ArcTan}\left[\frac{\sqrt{1-c x}}{\sqrt{1+c x}}\right]\right)^3\text{ArcTanh}\left[1-\frac{2}{1+\frac{i\sqrt{1-c x}}{\sqrt{1+c x}}}\right]}{c}+\frac{3\,i\,b\left(a+b\,\text{ArcTan}\left[\frac{\sqrt{1-c x}}{\sqrt{1+c x}}\right]\right)^2\text{PolyLog}\left[2,1-\frac{2}{1+\frac{i\sqrt{1-c x}}{\sqrt{1+c x}}}\right]}{2c}-\frac{3\,i\,b\left(a+b\,\text{ArcTan}\left[\frac{\sqrt{1-c x}}{\sqrt{1+c x}}\right]\right)^2\text{PolyLog}\left[2,-1+\frac{2}{1+\frac{i\sqrt{1-c x}}{\sqrt{1+c x}}}\right]}{2c}+\frac{3b^2\left(a+b\,\text{ArcTan}\left[\frac{\sqrt{1-c x}}{\sqrt{1+c x}}\right]\right)\text{PolyLog}\left[3,1-\frac{2}{1+\frac{i\sqrt{1-c x}}{\sqrt{1+c x}}}\right]}{2c}-\frac{3b^2\left(a+b\,\text{ArcTan}\left[\frac{\sqrt{1-c x}}{\sqrt{1+c x}}\right]\right)\text{PolyLog}\left[3,-1+\frac{2}{1+\frac{i\sqrt{1-c x}}{\sqrt{1+c x}}}\right]}{2c}-\frac{3\,i\,b^3\,\text{PolyLog}\left[4,1-\frac{2}{1+\frac{i\sqrt{1-c x}}{\sqrt{1+c x}}}\right]}{4c}+\frac{3\,i\,b^3\,\text{PolyLog}\left[4,-1+\frac{2}{1+\frac{i\sqrt{1-c x}}{\sqrt{1+c x}}}\right]}{4c}$$

Result (type 8, 42 leaves) :

$$\int\frac{\left(a+b\,\text{ArcTan}\left[\frac{\sqrt{1-c x}}{\sqrt{1+c x}}\right]\right)^3}{1-c^2x^2}\,dx$$

Problem 33: Unable to integrate problem.

$$\int\frac{\left(a+b\,\text{ArcTan}\left[\frac{\sqrt{1-c x}}{\sqrt{1+c x}}\right]\right)^2}{1-c^2x^2}\,dx$$

Optimal (type 4, 283 leaves, 7 steps) :

$$-\frac{2\left(a+b\,\text{ArcTan}\left[\frac{\sqrt{1-c x}}{\sqrt{1+c x}}\right]\right)^2\text{ArcTanh}\left[1-\frac{2}{1+\frac{i\sqrt{1-c x}}{\sqrt{1+c x}}}\right]}{c}+\frac{i b\left(a+b\,\text{ArcTan}\left[\frac{\sqrt{1-c x}}{\sqrt{1+c x}}\right]\right)\text{PolyLog}\left[2,1-\frac{2}{1+\frac{i\sqrt{1-c x}}{\sqrt{1+c x}}}\right]}{c}-\frac{i b\left(a+b\,\text{ArcTan}\left[\frac{\sqrt{1-c x}}{\sqrt{1+c x}}\right]\right)\text{PolyLog}\left[2,-1+\frac{2}{1+\frac{i\sqrt{1-c x}}{\sqrt{1+c x}}}\right]}{c}+\frac{b^2\,\text{PolyLog}\left[3,1-\frac{2}{1+\frac{i\sqrt{1-c x}}{\sqrt{1+c x}}}\right]}{2c}-\frac{b^2\,\text{PolyLog}\left[3,-1+\frac{2}{1+\frac{i\sqrt{1-c x}}{\sqrt{1+c x}}}\right]}{2c}$$

Result (type 8, 42 leaves):

$$\int\frac{\left(a+b\,\text{ArcTan}\left[\frac{\sqrt{1-c x}}{\sqrt{1+c x}}\right]\right)^2}{1-c^2x^2}\,dx$$

Problem 50: Result more than twice size of optimal antiderivative.

$$\int\text{ArcTan}[c+d\,\text{Tan}[a+b x]]\,dx$$

Optimal (type 4, 198 leaves, 7 steps):

$$x\,\text{ArcTan}[c+d\,\text{Tan}[a+b x]]+\frac{1}{2}i x\,\text{Log}\left[1+\frac{(1+i c+d)\,e^{2 i a+2 i b x}}{1+i c-d}\right]-\frac{1}{2}i x\,\text{Log}\left[1+\frac{(c+i(1-d))\,e^{2 i a+2 i b x}}{c+i(1+d)}\right]+\frac{\text{PolyLog}\left[2,-\frac{(1+i c+d)\,e^{2 i a+2 i b x}}{1+i c-d}\right]}{4b}-\frac{\text{PolyLog}\left[2,-\frac{(c+i(1-d))\,e^{2 i a+2 i b x}}{c+i(1+d)}\right]}{4b}$$

Result (type 4, 418 leaves):

$$x\,\text{ArcTan}[c+d\,\text{Tan}[a+b x]]+\frac{1}{4b}\left(2a\,\text{ArcTan}\left[\frac{c\left(1+e^{2i(a+b x)}\right)}{1+d+e^{2i(a+b x)}-d\,e^{2i(a+b x)}}\right]+2a\,\text{ArcTan}\left[\frac{c\left(1+e^{2i(a+b x)}\right)}{1+e^{2i(a+b x)}+d\left(-1+e^{2i(a+b x)}\right)}\right]+2i(a+b x)\,\text{Log}\left[1+\frac{(c-i(1+d))\,e^{2i(a+b x)}}{c+i(-1+d)}\right]-2i(a+b x)\,\text{Log}\left[1+\frac{(i+c-i d)\,e^{2i(a+b x)}}{c+i(1+d)}\right]+i a\,\text{Log}\left[e^{-4i(a+b x)}\left(c^2\left(1+e^{2i(a+b x)}\right)^2+\left(1+d+e^{2i(a+b x)}-d\,e^{2i(a+b x)}\right)^2\right)\right]-i a\,\text{Log}\left[e^{-4i(a+b x)}\left(c^2\left(1+e^{2i(a+b x)}\right)^2+\left(1+e^{2i(a+b x)}+d\left(-1+e^{2i(a+b x)}\right)\right)^2\right)\right]+\text{PolyLog}\left[2,-\frac{(c-i(1+d))\,e^{2i(a+b x)}}{c+i(-1+d)}\right]-\text{PolyLog}\left[2,-\frac{(i+c-i d)\,e^{2i(a+b x)}}{c+i(1+d)}\right]\right)$$

Problem 63: Result more than twice size of optimal antiderivative.

$$\int \text{ArcTan}[c + d \,\text{Cot}[a + b\, x]]\, dx$$

Optimal (type 4, 198 leaves, 7 steps):

$$x\,\text{ArcTan}[c + d\,\text{Cot}[a + b\,x]] + \frac{1}{2}\, i\, x\, \text{Log}\left[1 - \frac{(1 + i\, c - d)\, e^{2 i a + 2 i b x}}{1 + i\, c + d}\right] - \frac{1}{2}\, i\, x\, \text{Log}\left[1 - \frac{(c + i\,(1 + d))\, e^{2 i a + 2 i b x}}{c + i\,(1 - d)}\right] + \frac{\text{PolyLog}\left[2, \frac{(1+i\,c-d)\, e^{2 i a+2 i b x}}{1+i\,c+d}\right]}{4\,b} - \frac{\text{PolyLog}\left[2, \frac{(c+i\,(1+d))\, e^{2 i a+2 i b x}}{c+i\,(1-d)}\right]}{4\,b}$$

Result (type 4, 416 leaves):

$$x\,\text{ArcTan}[c + d\,\text{Cot}[a + b\,x]] + \frac{1}{4\,b}\left(2\,a\,\text{ArcTan}\left[\frac{c\,\left(-1 + e^{-2 i (a+b x)}\right)}{-1 + d + e^{-2 i (a+b x)} + d\, e^{-2 i (a+b x)}}\right] + 2\,a\,\text{ArcTan}\left[\frac{c\,\left(-1 + e^{2 i (a+b x)}\right)}{-1 + d + e^{2 i (a+b x)} + d\, e^{2 i (a+b x)}}\right] + 2\, i\, (a + b\,x)\,\text{Log}\left[1 - \frac{(c + i\,(-1 + d))\, e^{2 i (a+b x)}}{c - i\,(1 + d)}\right] - 2\, i\,(a + b\,x)\,\text{Log}\left[1 - \frac{(c + i\,(1 + d))\, e^{2 i (a+b x)}}{i + c - i\, d}\right] - i\, a\,\text{Log}\left[e^{-4 i (a+b x)}\left(c^2\left(-1 + e^{2 i (a+b x)}\right)^2 + \left(1 + d - e^{2 i (a+b x)} + d\, e^{2 i (a+b x)}\right)^2\right)\right] + i\, a\,\text{Log}\left[e^{-4 i (a+b x)}\left(c^2\left(-1 + e^{2 i (a+b x)}\right)^2 + \left(-1 + d + e^{2 i (a+b x)} + d\, e^{2 i (a+b x)}\right)^2\right)\right] + \text{PolyLog}\left[2, \frac{(c + i\,(-1 + d))\, e^{2 i (a+b x)}}{c - i\,(1 + d)}\right] - \text{PolyLog}\left[2, \frac{(c + i\,(1 + d))\, e^{2 i (a+b x)}}{i + c - i\, d}\right]\right)$$

Problem 75: Result more than twice size of optimal antiderivative.

$$\int x^2\, \text{ArcTan}[\text{Sinh}[x]]\, dx$$

Optimal (type 4, 108 leaves, 10 steps):

$$-\frac{2}{3}\, x^3\, \text{ArcTan}\left[e^x\right] + \frac{1}{3}\, x^3\, \text{ArcTan}[\text{Sinh}[x]] + i\, x^2\, \text{PolyLog}\left[2, -i\, e^x\right] - i\, x^2\, \text{PolyLog}\left[2, i\, e^x\right] - 2\, i\, x\, \text{PolyLog}\left[3, -i\, e^x\right] + 2\, i\, x\, \text{PolyLog}\left[3, i\, e^x\right] + 2\, i\, \text{PolyLog}\left[4, -i\, e^x\right] - 2\, i\, \text{PolyLog}\left[4, i\, e^x\right]$$

Result (type 4, 356 leaves):

$$\frac{1}{192} i \left(7 \pi^4 + 8 i \pi^3 x + 24 \pi^2 x^2 - 32 i \pi x^3 - 16 x^4 - 64 i x^3 \text{ArcTan}[\text{Sinh}[x]] + 8 i \pi^3 \text{Log}[1 + i e^{-x}] + 48 \pi^2 x \text{Log}[1 + i e^{-x}] - 96 i \pi x^2 \text{Log}[1 + i e^{-x}] - 64 x^3 \text{Log}[1 + i e^{-x}] - 48 \pi^2 x \text{Log}[1 - i e^x] + 96 i \pi x^2 \text{Log}[1 - i e^x] - 8 i \pi^3 \text{Log}[1 + i e^x] + 64 x^3 \text{Log}[1 + i e^x] + 8 i \pi^3 \text{Log}\left[\text{Tan}\left[\frac{1}{4} (\pi + 2 i x)\right]\right] - 48 (\pi - 2 i x)^2 \text{PolyLog}[2, -i e^{-x}] + 192 x^2 \text{PolyLog}[2, -i e^x] - 48 \pi^2 \text{PolyLog}[2, i e^x] + 192 i \pi x \text{PolyLog}[2, i e^x] + 192 i \pi \text{PolyLog}[3, -i e^{-x}] + 384 x \text{PolyLog}[3, -i e^{-x}] - 384 x \text{PolyLog}[3, -i e^x] - 192 i \pi \text{PolyLog}[3, i e^x] + 384 \text{PolyLog}[4, -i e^{-x}] + 384 \text{PolyLog}[4, -i e^x]\right)$$

Problem 76: Result more than twice size of optimal antiderivative.

$$\int (e + f x)^3 \text{ArcTan}[\text{Tanh}[a + b x]] \, dx$$

Optimal (type 4, 299 leaves, 12 steps):

$$-\frac{(e + f x)^4 \text{ArcTan}[e^{2 a+2 b x}]}{4 f} + \frac{(e + f x)^4 \text{ArcTan}[\text{Tanh}[a + b x]]}{4 f} + \frac{i (e + f x)^3 \text{PolyLog}[2, -i e^{2 a+2 b x}]}{4 b} - \frac{i (e + f x)^3 \text{PolyLog}[2, i e^{2 a+2 b x}]}{4 b} - \frac{3 i f (e + f x)^2 \text{PolyLog}[3, -i e^{2 a+2 b x}]}{8 b^2} + \frac{3 i f (e + f x)^2 \text{PolyLog}[3, i e^{2 a+2 b x}]}{8 b^2} + \frac{3 i f^2 (e + f x) \text{PolyLog}[4, -i e^{2 a+2 b x}]}{8 b^3} - \frac{3 i f^2 (e + f x) \text{PolyLog}[4, i e^{2 a+2 b x}]}{8 b^3} - \frac{3 i f^3 \text{PolyLog}[5, -i e^{2 a+2 b x}]}{16 b^4} + \frac{3 i f^3 \text{PolyLog}[5, i e^{2 a+2 b x}]}{16 b^4}$$

Result (type 4, 600 leaves):

$$\frac{1}{4} x (4 e^3 + 6 e^2 f x + 4 e f^2 x^2 + f^3 x^3) \text{ArcTan}[\text{Tanh}[a + b x]] - \frac{1}{16 b^4} i \left(8 b^4 e^3 x \text{Log}[1 - i e^{2 (a+b x)}] + 12 b^4 e^2 f x^2 \text{Log}[1 - i e^{2 (a+b x)}] + 8 b^4 e f^2 x^3 \text{Log}[1 - i e^{2 (a+b x)}] + 2 b^4 f^3 x^4 \text{Log}[1 - i e^{2 (a+b x)}] - 8 b^4 e^3 x \text{Log}[1 + i e^{2 (a+b x)}] - 12 b^4 e^2 f x^2 \text{Log}[1 + i e^{2 (a+b x)}] - 8 b^4 e f^2 x^3 \text{Log}[1 + i e^{2 (a+b x)}] - 2 b^4 f^3 x^4 \text{Log}[1 + i e^{2 (a+b x)}] - 4 b^3 (e + f x)^3 \text{PolyLog}[2, -i e^{2 (a+b x)}] + 4 b^3 (e + f x)^3 \text{PolyLog}[2, i e^{2 (a+b x)}] + 6 b^2 e^2 f \text{PolyLog}[3, -i e^{2 (a+b x)}] + 12 b^2 e f^2 x \text{PolyLog}[3, -i e^{2 (a+b x)}] + 6 b^2 f^3 x^2 \text{PolyLog}[3, -i e^{2 (a+b x)}] - 6 b^2 e^2 f \text{PolyLog}[3, i e^{2 (a+b x)}] - 12 b^2 e f^2 x \text{PolyLog}[3, i e^{2 (a+b x)}] - 6 b^2 f^3 x^2 \text{PolyLog}[3, i e^{2 (a+b x)}] - 6 b e f^2 \text{PolyLog}[4, -i e^{2 (a+b x)}] - 6 b f^3 x \text{PolyLog}[4, -i e^{2 (a+b x)}] + 6 b e f^2 \text{PolyLog}[4, i e^{2 (a+b x)}] + 6 b f^3 x \text{PolyLog}[4, i e^{2 (a+b x)}] + 3 f^3 \text{PolyLog}[5, -i e^{2 (a+b x)}] - 3 f^3 \text{PolyLog}[5, i e^{2 (a+b x)}]\right)$$

Problem 83: Result more than twice size of optimal antiderivative.

$$\int \text{ArcTan}[c+d\,\text{Tanh}[a+b\,x]]\,dx$$

Optimal (type 4, 174 leaves, 7 steps):

$$x\,\text{ArcTan}[c+d\,\text{Tanh}[a+b\,x]]+\frac{1}{2}\,i\,x\,\text{Log}\left[1+\frac{(i-c-d)\,e^{2a+2bx}}{i-c+d}\right]-\frac{1}{2}\,i\,x\,\text{Log}\left[1+\frac{(i+c+d)\,e^{2a+2bx}}{i+c-d}\right]+\frac{i\,\text{PolyLog}\left[2,-\frac{(i-c-d)\,e^{2a+2bx}}{i-c+d}\right]}{4\,b}-\frac{i\,\text{PolyLog}\left[2,-\frac{(i+c+d)\,e^{2a+2bx}}{i+c-d}\right]}{4\,b}$$

Result (type 4, 365 leaves):

$$x\,\text{ArcTan}[c+d\,\text{Tanh}[a+b\,x]]+\frac{1}{2\,b}\,i\left(2\,i\,a\,\text{ArcTan}\left[\frac{1+e^{2\,(a+b\,x)}}{c-d+c\,e^{2\,(a+b\,x)}+d\,e^{2\,(a+b\,x)}}\right]+(a+b\,x)\,\text{Log}\left[1-\frac{\sqrt{-i+c+d}\;e^{a+b\,x}}{\sqrt{i-c+d}}\right]+(a+b\,x)\,\text{Log}\left[1+\frac{\sqrt{-i+c+d}\;e^{a+b\,x}}{\sqrt{i-c+d}}\right]-(a+b\,x)\,\text{Log}\left[1-\frac{\sqrt{i+c+d}\;e^{a+b\,x}}{\sqrt{-i-c+d}}\right]-(a+b\,x)\,\text{Log}\left[1+\frac{\sqrt{i+c+d}\;e^{a+b\,x}}{\sqrt{-i-c+d}}\right]+\text{PolyLog}\left[2,-\frac{\sqrt{-i+c+d}\;e^{a+b\,x}}{\sqrt{i-c+d}}\right]+\text{PolyLog}\left[2,\frac{\sqrt{-i+c+d}\;e^{a+b\,x}}{\sqrt{i-c+d}}\right]-\text{PolyLog}\left[2,-\frac{\sqrt{i+c+d}\;e^{a+b\,x}}{\sqrt{-i-c+d}}\right]-\text{PolyLog}\left[2,\frac{\sqrt{i+c+d}\;e^{a+b\,x}}{\sqrt{-i-c+d}}\right]\right)$$

Problem 93: Result more than twice size of optimal antiderivative.

$$\int (e+f\,x)^3\,\text{ArcTan}[\text{Coth}[a+b\,x]]\,dx$$

Optimal (type 4, 299 leaves, 12 steps):

$$\frac{(e+f\,x)^4\,\text{ArcTan}\left[e^{2a+2bx}\right]}{4\,f}+\frac{(e+f\,x)^4\,\text{ArcTan}[\text{Coth}[a+b\,x]]}{4\,f}-\frac{i\,(e+f\,x)^3\,\text{PolyLog}\left[2,-i\,e^{2a+2bx}\right]}{4\,b}+\frac{i\,(e+f\,x)^3\,\text{PolyLog}\left[2,i\,e^{2a+2bx}\right]}{4\,b}+\frac{3\,i\,f\,(e+f\,x)^2\,\text{PolyLog}\left[3,-i\,e^{2a+2bx}\right]}{8\,b^2}-\frac{3\,i\,f\,(e+f\,x)^2\,\text{PolyLog}\left[3,i\,e^{2a+2bx}\right]}{8\,b^2}-\frac{3\,i\,f^2\,(e+f\,x)\,\text{PolyLog}\left[4,-i\,e^{2a+2bx}\right]}{8\,b^3}+\frac{3\,i\,f^2\,(e+f\,x)\,\text{PolyLog}\left[4,i\,e^{2a+2bx}\right]}{8\,b^3}+\frac{3\,i\,f^3\,\text{PolyLog}\left[5,-i\,e^{2a+2bx}\right]}{16\,b^4}-\frac{3\,i\,f^3\,\text{PolyLog}\left[5,i\,e^{2a+2bx}\right]}{16\,b^4}$$

Result (type 4, 600 leaves):

$$\frac{1}{4} x \left(4 e^3 + 6 e^2 f x + 4 e f^2 x^2 + f^3 x^3\right) \text{ArcTan}[\text{Coth}[a + b x]] + \frac{1}{16 b^4} i \left(8 b^4 e^3 x \text{Log}\left[1 - i e^{2 (a+b x)}\right] + 12 b^4 e^2 f x^2 \text{Log}\left[1 - i e^{2 (a+b x)}\right] + 8 b^4 e f^2 x^3 \text{Log}\left[1 - i e^{2 (a+b x)}\right] + 2 b^4 f^3 x^4 \text{Log}\left[1 - i e^{2 (a+b x)}\right] - 8 b^4 e^3 x \text{Log}\left[1 + i e^{2 (a+b x)}\right] - 12 b^4 e^2 f x^2 \text{Log}\left[1 + i e^{2 (a+b x)}\right] - 8 b^4 e f^2 x^3 \text{Log}\left[1 + i e^{2 (a+b x)}\right] - 2 b^4 f^3 x^4 \text{Log}\left[1 + i e^{2 (a+b x)}\right] - 4 b^3 (e + f x)^3 \text{PolyLog}\left[2, -i e^{2 (a+b x)}\right] + 4 b^3 (e + f x)^3 \text{PolyLog}\left[2, i e^{2 (a+b x)}\right] + 6 b^2 e^2 f \text{PolyLog}\left[3, -i e^{2 (a+b x)}\right] + 12 b^2 e f^2 x \text{PolyLog}\left[3, -i e^{2 (a+b x)}\right] + 6 b^2 f^3 x^2 \text{PolyLog}\left[3, -i e^{2 (a+b x)}\right] - 6 b^2 e^2 f \text{PolyLog}\left[3, i e^{2 (a+b x)}\right] - 12 b^2 e f^2 x \text{PolyLog}\left[3, i e^{2 (a+b x)}\right] - 6 b^2 f^3 x^2 \text{PolyLog}\left[3, i e^{2 (a+b x)}\right] - 6 b e f^2 \text{PolyLog}\left[4, -i e^{2 (a+b x)}\right] - 6 b f^3 x \text{PolyLog}\left[4, -i e^{2 (a+b x)}\right] + 6 b e f^2 \text{PolyLog}\left[4, i e^{2 (a+b x)}\right] + 6 b f^3 x \text{PolyLog}\left[4, i e^{2 (a+b x)}\right] + 3 f^3 \text{PolyLog}\left[5, -i e^{2 (a+b x)}\right] - 3 f^3 \text{PolyLog}\left[5, i e^{2 (a+b x)}\right]\right)$$

Problem 100: Result more than twice size of optimal antiderivative.

$$\int \text{ArcTan}[c + d \text{Coth}[a + b x]] \, dx$$

Optimal (type 4, 174 leaves, 7 steps):

$$x \text{ArcTan}[c + d \text{Coth}[a + b x]] + \frac{1}{2} i x \text{Log}\left[1 - \frac{(i - c - d) e^{2 a + 2 b x}}{i - c + d}\right] - \frac{1}{2} i x \text{Log}\left[1 - \frac{(i + c + d) e^{2 a + 2 b x}}{i + c - d}\right] + \frac{i \text{PolyLog}\left[2, \frac{(i-c-d) e^{2 a+2 b x}}{i-c+d}\right]}{4 b} - \frac{i \text{PolyLog}\left[2, \frac{(i+c+d) e^{2 a+2 b x}}{i+c-d}\right]}{4 b}$$

Result (type 4, 365 leaves):

$$x \text{ArcTan}[c + d \text{Coth}[a + b x]] + \frac{1}{2 b} i \left(2 i a \text{ArcTan}\left[\frac{-1 + e^{2 (a+b x)}}{-c + d + c e^{2 (a+b x)} + d e^{2 (a+b x)}}\right] + (a + b x) \text{Log}\left[1 - \frac{\sqrt{-i + c + d}\, e^{a+b x}}{\sqrt{-i + c - d}}\right] + (a + b x) \text{Log}\left[1 + \frac{\sqrt{-i + c + d}\, e^{a+b x}}{\sqrt{-i + c - d}}\right] - (a + b x) \text{Log}\left[1 - \frac{\sqrt{i + c + d}\, e^{a+b x}}{\sqrt{i + c - d}}\right] - (a + b x) \text{Log}\left[1 + \frac{\sqrt{i + c + d}\, e^{a+b x}}{\sqrt{i + c - d}}\right] + \text{PolyLog}\left[2, -\frac{\sqrt{-i + c + d}\, e^{a+b x}}{\sqrt{-i + c - d}}\right] + \text{PolyLog}\left[2, \frac{\sqrt{-i + c + d}\, e^{a+b x}}{\sqrt{-i + c - d}}\right] - \text{PolyLog}\left[2, -\frac{\sqrt{i + c + d}\, e^{a+b x}}{\sqrt{i + c - d}}\right] - \text{PolyLog}\left[2, \frac{\sqrt{i + c + d}\, e^{a+b x}}{\sqrt{i + c - d}}\right]\right)$$

Problem 116: Attempted integration timed out after 120 seconds.

$$\int \text{ArcTan}\left[a + b f^{c+d x}\right] dx$$

Optimal (type 4, 196 leaves, 6 steps):

$$-\frac{\text{ArcTan}\left[a+b\,f^{c+d\,x}\right]\,\text{Log}\left[\frac{2}{1-i\,\left(a+b\,f^{c+d\,x}\right)}\right]}{d\,\text{Log}[f]}+\frac{\text{ArcTan}\left[a+b\,f^{c+d\,x}\right]\,\text{Log}\left[\frac{2\,b\,f^{c+d\,x}}{(i-a)\,\left(1-i\,\left(a+b\,f^{c+d\,x}\right)\right)}\right]}{d\,\text{Log}[f]}+\frac{i\,\text{PolyLog}\left[2,\,1-\frac{2}{1-i\,\left(a+b\,f^{c+d\,x}\right)}\right]}{2\,d\,\text{Log}[f]}-\frac{i\,\text{PolyLog}\left[2,\,1-\frac{2\,b\,f^{c+d\,x}}{(i-a)\,\left(1-i\,\left(a+b\,f^{c+d\,x}\right)\right)}\right]}{2\,d\,\text{Log}[f]}$$

Result (type 1, 1 leaves):

???

Problem 117: Unable to integrate problem.

$$\int x\,\text{ArcTan}\left[a+b\,f^{c+d\,x}\right]\,dx$$

Optimal (type 4, 232 leaves, 9 steps):

$$\frac{1}{2}\,x^2\,\text{ArcTan}\left[a+b\,f^{c+d\,x}\right]-\frac{1}{4}\,i\,x^2\,\text{Log}\left[1-\frac{i\,b\,f^{c+d\,x}}{1-i\,a}\right]+\frac{1}{4}\,i\,x^2\,\text{Log}\left[1+\frac{i\,b\,f^{c+d\,x}}{1+i\,a}\right]-\frac{i\,x\,\text{PolyLog}\left[2,\,\frac{i\,b\,f^{c+d\,x}}{1-i\,a}\right]}{2\,d\,\text{Log}[f]}+\frac{i\,x\,\text{PolyLog}\left[2,\,-\frac{i\,b\,f^{c+d\,x}}{1+i\,a}\right]}{2\,d\,\text{Log}[f]}+\frac{i\,\text{PolyLog}\left[3,\,\frac{i\,b\,f^{c+d\,x}}{1-i\,a}\right]}{2\,d^2\,\text{Log}[f]^2}-\frac{i\,\text{PolyLog}\left[3,\,-\frac{i\,b\,f^{c+d\,x}}{1+i\,a}\right]}{2\,d^2\,\text{Log}[f]^2}$$

Result (type 8, 16 leaves):

$$\int x\,\text{ArcTan}\left[a+b\,f^{c+d\,x}\right]\,dx$$

Problem 118: Unable to integrate problem.

$$\int x^2\,\text{ArcTan}\left[a+b\,f^{c+d\,x}\right]\,dx$$

Optimal (type 4, 302 leaves, 11 steps):

$$\frac{1}{3}\,x^3\,\text{ArcTan}\left[a+b\,f^{c+d\,x}\right]-\frac{1}{6}\,i\,x^3\,\text{Log}\left[1-\frac{i\,b\,f^{c+d\,x}}{1-i\,a}\right]+\frac{1}{6}\,i\,x^3\,\text{Log}\left[1+\frac{i\,b\,f^{c+d\,x}}{1+i\,a}\right]-\frac{i\,x^2\,\text{PolyLog}\left[2,\,\frac{i\,b\,f^{c+d\,x}}{1-i\,a}\right]}{2\,d\,\text{Log}[f]}+\frac{i\,x^2\,\text{PolyLog}\left[2,\,-\frac{i\,b\,f^{c+d\,x}}{1+i\,a}\right]}{2\,d\,\text{Log}[f]}+\frac{i\,x\,\text{PolyLog}\left[3,\,\frac{i\,b\,f^{c+d\,x}}{1-i\,a}\right]}{d^2\,\text{Log}[f]^2}-\frac{i\,x\,\text{PolyLog}\left[3,\,-\frac{i\,b\,f^{c+d\,x}}{1+i\,a}\right]}{d^2\,\text{Log}[f]^2}-\frac{i\,\text{PolyLog}\left[4,\,\frac{i\,b\,f^{c+d\,x}}{1-i\,a}\right]}{d^3\,\text{Log}[f]^3}+\frac{i\,\text{PolyLog}\left[4,\,-\frac{i\,b\,f^{c+d\,x}}{1+i\,a}\right]}{d^3\,\text{Log}[f]^3}$$

Result (type 8, 18 leaves):

$$\int x^2\,\text{ArcTan}\left[a+b\,f^{c+d\,x}\right]\,dx$$

Problem 148: Result is not expressed in closed-form.

$$\int e^{c\,(a+b\,x)}\,\text{ArcTan}[\text{Cosh}[a\,c+b\,c\,x]]\,dx$$

Optimal (type 3, 103 leaves, 8 steps):

$$\frac{e^{a\,c+b\,c\,x}\,\text{ArcTan}\left[\text{Cosh}\left[c\,\left(a+b\,x\right)\right]\right]}{b\,c}-\frac{\left(1-\sqrt{2}\right)\,\text{Log}\left[3-2\,\sqrt{2}+e^{2\,c\,(a+b\,x)}\right]}{2\,b\,c}-\frac{\left(1+\sqrt{2}\right)\,\text{Log}\left[3+2\,\sqrt{2}+e^{2\,c\,(a+b\,x)}\right]}{2\,b\,c}$$

Result (type 7, 146 leaves):

$$\frac{1}{2\,b\,c}\left(-4\,c\,\left(a+b\,x\right)+2\,e^{c\,(a+b\,x)}\,\text{ArcTan}\left[\frac{1}{2}\,e^{-c\,(a+b\,x)}\,\left(1+e^{2\,c\,(a+b\,x)}\right)\right]+\text{RootSum}\left[1+6\,\#1^2+\#1^4\,\&,\right.\right.$$
$$\left.\left.\frac{1}{1+3\,\#1^2}\left(a\,c+b\,c\,x-\text{Log}\left[e^{c\,(a+b\,x)}-\#1\right]+7\,a\,c\,\#1^2+7\,b\,c\,x\,\#1^2-7\,\text{Log}\left[e^{c\,(a+b\,x)}-\#1\right]\,\#1^2\right)\,\&\right]\right)$$

Problem 149: Result is not expressed in closed-form.

$$\int e^{c\,(a+b\,x)}\,\text{ArcTan}[\text{Tanh}[a\,c+b\,c\,x]]\,dx$$

Optimal (type 3, 180 leaves, 13 steps):

$$\frac{\text{ArcTan}\left[1-\sqrt{2}\,e^{a\,c+b\,c\,x}\right]}{\sqrt{2}\,b\,c}-\frac{\text{ArcTan}\left[1+\sqrt{2}\,e^{a\,c+b\,c\,x}\right]}{\sqrt{2}\,b\,c}+\frac{e^{a\,c+b\,c\,x}\,\text{ArcTan}\left[\text{Tanh}\left[c\,\left(a+b\,x\right)\right]\right]}{b\,c}-$$
$$\frac{\text{Log}\left[1+e^{2\,c\,(a+b\,x)}-\sqrt{2}\,e^{a\,c+b\,c\,x}\right]}{2\,\sqrt{2}\,b\,c}+\frac{\text{Log}\left[1+e^{2\,c\,(a+b\,x)}+\sqrt{2}\,e^{a\,c+b\,c\,x}\right]}{2\,\sqrt{2}\,b\,c}$$

Result (type 7, 89 leaves):

$$\frac{1}{2\,b\,c}\left(2\,e^{c\,(a+b\,x)}\,\text{ArcTan}\left[\frac{-1+e^{2\,c\,(a+b\,x)}}{1+e^{2\,c\,(a+b\,x)}}\right]+\text{RootSum}\left[1+\#1^4\,\&,\,\frac{a\,c+b\,c\,x-\text{Log}\left[e^{c\,(a+b\,x)}-\#1\right]}{\#1}\,\&\right]\right)$$

Problem 150: Result is not expressed in closed-form.

$$\int e^{c\,(a+b\,x)}\,\text{ArcTan}[\text{Coth}[a\,c+b\,c\,x]]\,dx$$

Optimal (type 3, 180 leaves, 13 steps):

$$-\frac{\text{ArcTan}\left[1-\sqrt{2}\,e^{a\,c+b\,c\,x}\right]}{\sqrt{2}\,b\,c}+\frac{\text{ArcTan}\left[1+\sqrt{2}\,e^{a\,c+b\,c\,x}\right]}{\sqrt{2}\,b\,c}+\frac{e^{a\,c+b\,c\,x}\,\text{ArcTan}\left[\text{Coth}\left[c\,\left(a+b\,x\right)\right]\right]}{b\,c}+$$
$$\frac{\text{Log}\left[1+e^{2\,c\,(a+b\,x)}-\sqrt{2}\,e^{a\,c+b\,c\,x}\right]}{2\,\sqrt{2}\,b\,c}-\frac{\text{Log}\left[1+e^{2\,c\,(a+b\,x)}+\sqrt{2}\,e^{a\,c+b\,c\,x}\right]}{2\,\sqrt{2}\,b\,c}$$

Result (type 7, 89 leaves):

$$\frac{1}{2\,b\,c}\left(2\,e^{c\,(a+b\,x)}\,\text{ArcTan}\left[\frac{1+e^{2\,c\,(a+b\,x)}}{-1+e^{2\,c\,(a+b\,x)}}\right]+\text{RootSum}\left[1+\#1^4\,\&,\,\frac{-a\,c-b\,c\,x+\text{Log}\left[e^{c\,(a+b\,x)}-\#1\right]}{\#1}\,\&\right]\right)$$

Problem 151: Result is not expressed in closed-form.

$$\int e^{c\,(a+b\,x)}\,\text{ArcTan}[\text{Sech}[a\,c+b\,c\,x]]\,dx$$

Optimal (type 3, 103 leaves, 8 steps):

$$\frac{e^{a\,c+b\,c\,x}\,\text{ArcTan}\left[\text{Sech}\left[c\,(a+b\,x)\right]\right]}{b\,c}+\frac{\left(1-\sqrt{2}\right)\,\text{Log}\left[3-2\,\sqrt{2}+e^{2\,c\,(a+b\,x)}\right]}{2\,b\,c}+\frac{\left(1+\sqrt{2}\right)\,\text{Log}\left[3+2\,\sqrt{2}+e^{2\,c\,(a+b\,x)}\right]}{2\,b\,c}$$

Result (type 7, 145 leaves):

$$\frac{1}{2\,b\,c}\left(4\,c\,(a+b\,x)+2\,e^{c\,(a+b\,x)}\,\text{ArcTan}\left[\frac{2\,e^{c\,(a+b\,x)}}{1+e^{2\,c\,(a+b\,x)}}\right]+\text{RootSum}\left[1+6\,\#1^2+\#1^4\,\&,\right.\right.$$
$$\left.\left.\frac{1}{1+3\,\#1^2}\left(-a\,c-b\,c\,x+\text{Log}\left[e^{c\,(a+b\,x)}-\#1\right]-7\,a\,c\,\#1^2-7\,b\,c\,x\,\#1^2+7\,\text{Log}\left[e^{c\,(a+b\,x)}-\#1\right]\,\#1^2\right)\,\&\right]\right)$$

Problem 153: Result unnecessarily involves higher level functions.

$$\int \frac{(a+b\,\text{ArcTan}[c\,x^n])\,(d+e\,\text{Log}[f\,x^m])}{x}\,dx$$

Optimal (type 4, 163 leaves, 13 steps):

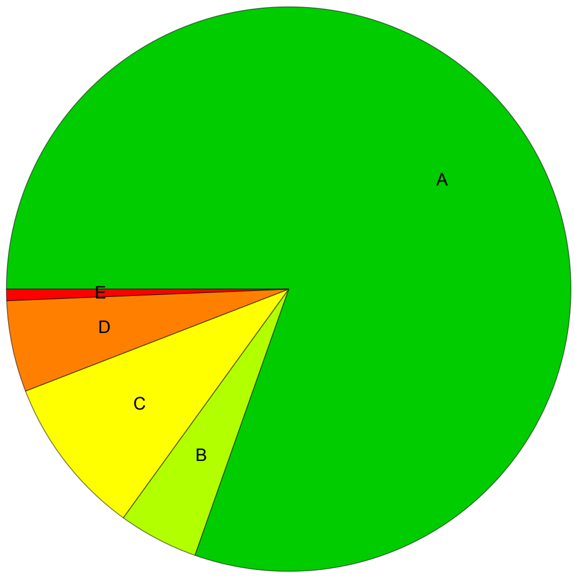
$$a\,d\,\text{Log}[x]+\frac{a\,e\,\text{Log}[f\,x^m]^2}{2\,m}+\frac{i\,b\,d\,\text{PolyLog}[2,-i\,c\,x^n]}{2\,n}+\frac{i\,b\,e\,\text{Log}[f\,x^m]\,\text{PolyLog}[2,-i\,c\,x^n]}{2\,n}-\frac{i\,b\,d\,\text{PolyLog}[2,i\,c\,x^n]}{2\,n}-\frac{i\,b\,e\,\text{Log}[f\,x^m]\,\text{PolyLog}[2,i\,c\,x^n]}{2\,n}-\frac{i\,b\,e\,m\,\text{PolyLog}[3,-i\,c\,x^n]}{2\,n^2}+\frac{i\,b\,e\,m\,\text{PolyLog}[3,i\,c\,x^n]}{2\,n^2}$$

Result (type 5, 116 leaves):

$$-\frac{1}{n^2}b\,c\,e\,m\,x^n\,\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{1}{2},\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2},\frac{3}{2}\right\},-c^2\,x^{2\,n}\right]+\frac{1}{n}b\,c\,x^n\,\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{1}{2},1\right\},\left\{\frac{3}{2},\frac{3}{2}\right\},-c^2\,x^{2\,n}\right]\,(d+e\,\text{Log}[f\,x^m])+\frac{1}{2}\,a\,\text{Log}[x]\,(2\,d-e\,m\,\text{Log}[x]+2\,e\,\text{Log}[f\,x^m])$$

Summary of Integration Test Results

153 integration problems



A - 123 optimal antiderivatives

B - 7 more than twice size of optimal antiderivatives

C - 14 unnecessarily complex antiderivatives

D - 8 unable to integrate problems

E - 1 integration timeouts